PHYSICS 534

ANSWERS

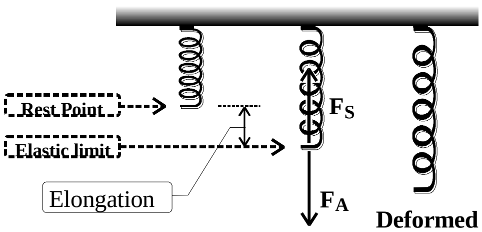
EXERCISE-42 **Hooke's Law (Springs)**

Clinton Davisson was awarded the Nobel prize for physics in 1937 for his work on the diffraction of electrons.

DAVISSON

A spring is a device that stores potential energy. When a spring is stretched (or compressed), a force is applied through a distance. Thus, work is done. The work done goes to the spring in the form of potential energy (E_P). The spring force is a restoring force which, in bringing the spring back to its rest point, does work.

Note that every spring has an elastic limit. If the spring is stretched within its elastic limit, it "springs" back to its "rest point". If a spring is stretched beyond its elastic limit, it becomes deformed. As illustrated below, the distance a spring is stretched is called the "elongation". Robert Hooke was the first to discover that the spring force is directly proportional to the elongation. Today, we call this law Hooke's Law.



Note that the spring force (F_S) is opposite the applied force (F_A).

The formula for the spring is: $F_S = kx$ ☜ (Hooke's Law)

where F_S is the spring force, in newtons (N)
k is the spring constant, in N/m
x is the elongation, in metres

The spring constant is different for different springs and depends upon the type of material the spring is made of as well as the thickness of the spring coil. The greater the value of the spring constant, the "stiffer" the spring.

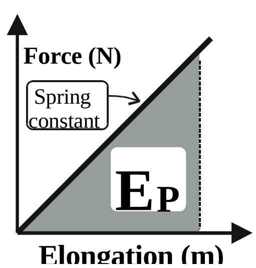
The formula for the potential energy stored in a spring is:

$$E_P = \tfrac{1}{2}kx^2$$

where E_P is the potential energy stored in the spring, in joules (J)
k is the spring constant, in N/m
x is the elongation, in metres

Remember that in an ideal spring, there is no loss of energy (E_P) due to friction.

When the spring force (F_S) is plotted versus the elongation (x) of the spring, the resulting graph is a linear relation. The slope of the curve represents the ***spring constant*** while the area under the curve represents the ***potential energy*** stored in the spring.



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1. What determines the spring constant (k) of a spring?

The type of material the coil is made from and the thickness of the coil.

2. A coiled spring is stretched 0.05 m by a weight of 0.50 N hung from one end.

a) How far will the spring stretch if a 1.0 N weight replaces the 0.50 N weight? [0.1 m]

$\underline{\text{First find the spring constant:}}$ $\because\ F_S = kx \quad \therefore\ k = F_S / x = 0.5\ N / 0.05\ m = 10\ N/m$

$\underline{\text{Now find how far the spring will stretch:}}$

$\because\ F_S = kx \quad \therefore\ x = \frac{F_S}{k} = \frac{1.0\ N}{10\ N/m} = 0.1\ m$

b) What weight will stretch the spring a distance of 0.03 m? [0.3 N]

$\underline{\text{Note}}$: The weight equals the spring force.

$\therefore\ F_S = kx = (10\ N/m)(0.03\ m) = 0.3\ N$

3. When the spring force is plotted versus the elongation of the spring:

a) How do you determine the spring constant (k) from the graph?

By finding the slope of the line plotted.

b) How do you determine the potential energy (E_P) stored in a spring from the graph?

By finding the area (from the curve to the x-axis) of the line plotted.

4. An ideal spring, whose spring constant is 14.0 N/m, is stretched 0.40 m when a mass of 0.560 kg is hung from it, how much potential energy (E_P) is stored in this spring? [1.12 J]

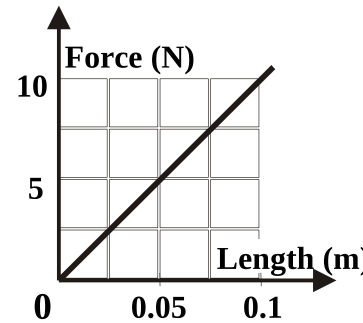
$$E_P = \frac{kx^2}{2} = \frac{(14\ N/s)(0.40)^2}{2} = 1.12\ J$$

5. The work done to compress a spring from 0 to 0.15 m is 8.0 J. How much work is required to compress this spring from 0 to 0.30 m? [32 J]

$\underline{\text{Find the value of } k}$: $\because\ E_P = \frac{kx^2}{2} \quad \therefore\ k = 2(E_P)/x^2 = 2(8.0\ J)/(0.15\ m)^2 = 711\ N/m$

$\underline{\text{Find the new } E_P}$: $\because\ E_P = \frac{kx^2}{2} = (711\ N/m)(0.30\ m)^2 = 32\ J$

6. The graph on the right represents the force-compression graph of an ideal spring.

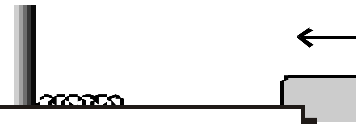


a) Determine the spring constant. [100 N/m]

$$\text{Slope} = \frac{10\ \text{N}}{0.1\ \text{m}} = 100\ \text{N/m}$$

Therefore, the spring constant (k) is 100 N/m.

b) Calculate the loss in kinetic energy of an object that collides with this spring and compresses it a distance of 0.60 m. [18 J]



Note: The loss in E_K of the object = gain in E_P of the spring.

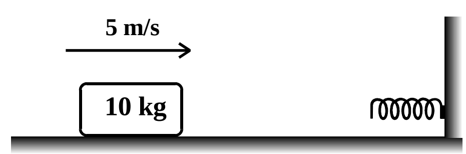
Thus: $E_P = \frac{kx^2}{2} = \frac{(100\ \text{N/m})(0.60\ \text{m})^2}{2} = 18\ \text{J}$

7. A spring is compressed 0.10 m when a force of 2.0 N is exerted upon it. What force must be used to compress this spring 0.40 meters? [8.0 N]

Find the spring constant: $\because\ F_S = kx \quad \therefore\ k = F_S / x = 2.0\ \text{N} / 0.10\ \text{m} = 20\ \text{N/m}$

Now find the spring force required: $F_S = kx = (20\ \text{N/m})(0.40\ \text{m}) = 8\ \text{N}$

8. A mass of 10 kg, having a velocity of 5 m/s, slides into a spring as illustrated in the diagram on the right. Assuming the system is frictionless, which of the following statements is ***correct***?



I- At maximum compression, the E_K of the mass is zero.

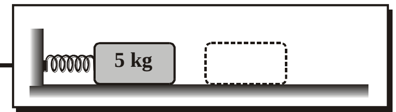
II- At maximum compression, the E_P of the spring equals the E_K of the mass.

III- After the interaction, the mass moves with a velocity of 5 m/s.

➔**a) I** b) II c) III d) I and III e) all are correct

Velocity = -5 m/s

9. A 5 kg block is forced against a horizontal spring a distance of 10 cm. When released, the block moves a horizontal distance of 2 cm then stops. If the spring constant of this spring is 150 N/m, what is the coefficient of friction between the block and the horizontal surface? [0.06]



Note: The spring force is the frictional force ($F_S = f$).

Find the spring force: $F_S = kx = (150\ \text{N/m})(0.02\ \text{m}) = 3\ \text{N}$

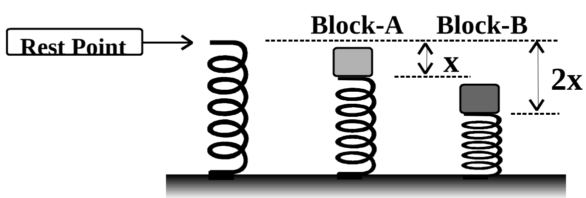
[redacted]

CAUTION: Do not confuse the spring constant with the coefficient of friction.

Find the coefficient of friction: $\because\ f = kF_N \qquad \therefore\ k = \dfrac{f}{F_N} = \dfrac{3\ \text{N}}{50\ \text{N}} = 0.06$

[redacted]

10. Block-A is placed on a spring and compresses it a distance x. As illustrated below, when Block-B is placed on the same spring, it is compressed a distance of 2x.

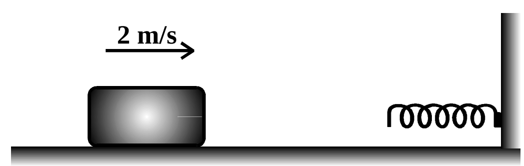


What is the value of the following ratio?

Potential energy of spring with Block-B on t
Potential energy of spring with Block-A on t

$$\frac{E_{P\text{ of }B}}{E_{P\text{ of }A}} = \frac{\frac{kx_B^2}{2}}{\frac{kx_A^2}{2}} = \frac{x_B^2}{x_A^2} = \frac{(2x)^2}{x^2} = \frac{4x^2}{x^2} = \frac{4}{1}$$

11. A block of unknown mass moves at 2 m/s towards a horizontal spring whose spring constant is 1000 N/m.



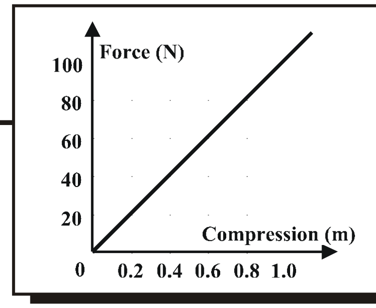
If the spring is compressed a maximum distance of 0.4 m, what is the mass of the block? [40 kg]

Note: At maximum compression, E_P = Initial E_K of mass.

Thus, $E_P = E_K$ or $\dfrac{kx^2}{2} = \dfrac{mv^2}{2}$

Therefore: $m = \dfrac{kx^2}{v^2} = \dfrac{(1000\ \text{N/m})(0.4\ \text{m})^2}{(2\ \text{m/s})^2} = 40\ \text{kg}$

12. The force-compression graph of a spring is illustrated on the right.



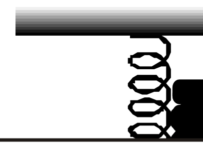
Note: From the given graph, the slope of the line is the spring constant.

$$\text{Thus, } k = \text{slope} = \frac{100\ \text{N}}{1.0\ \text{m}} = 100\ \text{N/m}$$

Now find the E_P of the spring which equals the E_K lost by the object.

$$E_P = \frac{kx^2}{2} = \frac{(100\ \text{N/m})(0.8\ \text{m})^2}{2} = 32\ \text{J}$$

13. When a 24 kg mass is attached to the end of a spring hanging vertically, the spring experiences an elongation of 5.0 cm. How much potential energy is stored in the spring? [6 J]



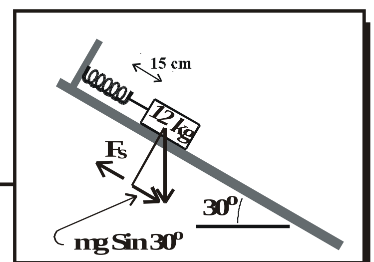
Find the spring constant:

$$\because\ F_S = kx \quad \therefore\ k = F_S / x = mg / x = (24\ \text{kg})(10\ \text{m/s}^2) / 0.05\ \text{m} = 4800\ \text{N/m}$$

Find the potential energy stored in the spring:

$$E_P = \frac{kx^2}{2} = \frac{(4800\ \text{N/m})(0.05\ \text{m})^2}{2} = 6\ \text{J}$$

14. A spring is fixed along an inclined plane whose angle of incline is 30^o as illustrated in the diagram on the right. A 12 kg block is attached to the spring thereby stretching it 15 cm. Find the spring constant of this spring. [400 N/m]



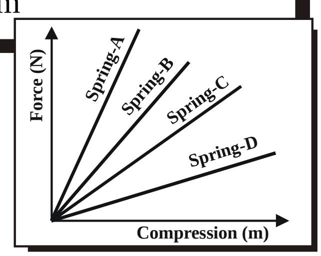
Note: Relative to the inclined plane, the spring force (F_S) equals the horizontal component of the weight ($mg \sin 30^o$).

$$\therefore\ F_S = mg \sin 30^o = (12\ \text{kg})(10\ \text{m/s}^2)\sin 30^o = 60\ \text{N}$$

Now find the spring constant:

$$\because\ F_S = kx \quad \therefore\ k = F_S / x = 60\ \text{N} / 0.15\ \text{m} = 400\ \text{N/m}$$

15. The graph on the left displays the force-compression curve of four springs labeled Spring-A, Spring-B, Spring-C and Spring D. Which spring is the "springiest"?



Spring-D **(Least slope)**

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